MI1046 Differential Equations and Series

Version: 2024.1.0

Objective: Differential Equations and Series is intended for engineering students and others who require a working knowledge of differential equations and series; included are techniques and applications of differential equations and infinite series. Since many physical laws and relations appear mathematically in the form of differential equations, such equations are of fundamental importance in engineering mathematics. Therefore, the main objective of this course is to help students to be familiar with various physical and geometrical problems that lead to differential equations. Moreover, it provides students with the most important standard methods for solving such equations.

Contents: Infinite series including number series and series of functions; Differential equations including first order equations and second order linear equations; Laplace transforms and their applications.

1. GENERAL INFORMATION

Course title:	**Differential Equations and Series**
Unit in charge:	Faculty of Mathematics and Informatics
Course ID:	MI1046
Course Units:	3(2-2-0-6) - Lectures: 30 hours - Exercises: 30 hours
Previous module:	- Calculus 1
Prerequisites:	- Calculus 1
Companion module:	None

2. DESCRIPTION: To provide students with basic knowledge about infinite series, to present various physical and geometrical problems that lead to differential equations, and to give students the most important standard methods for solving such equations, included are first order equations, second order linear equations, Laplace transforms and their applications.

3. OBJECTIVES AND EXPECTED OUTCOMES

Students who complete this module have the abilities to:

Objectives	Objectives description/Expected Outcomes	Outcome standard allocated for modules/ Levels (I/T/U)
[1]	[2]	[3]
M1	Students understand and can present in their own language the working knowledge of differential equations and series.	
M1.1	Students understand and can present the concepts of first	I/T/U

Objectives	Objectives description/Expected Outcomes	Outcome standard allocated for modules/ Levels (I/T/U)
	and linear second order differential equations and infinite series of numbers and functions.	
M1.2	Students can apply the techniques of solving differential equations and investigating the convergence and divergence infinite series and sums to concrete modelings and problems.	T/U
M2	Students are capable of using concepts and tools from differential equations and series to modelize, to analyze and solve the engineering and real life problems.	
M2.1	Students are able to modelize the problems in mechanics, electrical engineering using the tools from differential equations and series.	T/U
M2.2	Students are able to solve and analyze the modeling problems using knowledge and techniques from differential equations and series.	I/T/U
M2.3	Students are able to predict the behavior of engineering and real life problems using the results obtained in modeling through differential equations and series.	I/T

4. COURSE MATERIALS

Textbooks

[1] Nguyen Thieu Huy: Lecture on Infinite Series and Differential Equations, weblink: https://fami.hust.edu.vn/wp-content/uploads/CalculusIII.pdf

[2] Nguyen Thieu Huy, Vu Thi Ngoc Ha: *Infinite series and differential equations*, Hanoi University of Science and Technology, Elite Technology program, 2022.

References

[1] Glenn Ledder, *Differential Equations: a Modeling Approach*, McGraw-hill, New York, 2005.

[2] J. Stewart, D. Clegg, S. Watson, *Multivariable Calculus,* 9th Edition, Cengage Learning, 2020.

[3] W. E. Boyce, R.C. DiPrima, D.B. Meade, *Elementary Differential Equations and Boundary Value Problems,* 11th Edition, Wiley, 2017.

[4] R. Bronson, G. B. Costa, *Differential Equations,* 4th Edition, The McGraw-Hill, 2014.

5. ASSESSMENT

Components	Evaluation method	Description	Rated outcome standards	Proportion
[1]	**[2]**	**[3]**	**[4]**	**[5]**

A1. Attendance mark	**A1.** Learning attitude	Attendance check		**20%**
A2. Process mark (*)	**A2.1.** Midterm exam 1 (KT1 points on the 15-point scale) (Contents: from week 1 to week 5)	Multiple-choice	M1.1, M1.2, M2.1, M2.2, M2.3	**30%**
	A2.2. Midterm exam 2 (KT2 points on the 15-point scale) (Contents: from week 6 to week 10)			
A3. Final exam mark	**A3. Final exam**	Essay	M1.1, M1.2, M2.1, M2.2, M2.3	**50%**

(*) *The process mark is one third of the sum of the two midterm exams' marks. The process mark is adjusted by adding points for the performance of students during the course. These points vary from -1 to +1 according to the Rule of Faculty of Mathematics and Informatics together with the Regulations of Higher Education of Hanoi University of Science and Technology.*

6. COURSE PLAN

Week	Topics	Objective	Activities	Exercises
[1]	[2]	[3]	[4]	[5]
1	**PART ONE: INFINITE SERIES (13+13)** **I. Infinite series of numbers** 1. Infinite series, convergence and divergence, general terms, sums, partial sums, remainder. Fundamental facts 2. Tests for convergence and divergence +) Necessary condition for convergence, harmonic series, +) Comparison test, geometric series, +) Integral test, Riemann *p*-series	M1.1 M1.2 M2.1 M2.3	Lecturer: - Self-introduce - Introduce the course outline - Explain teaching and learning methods; and forms of subject assessment - Lecture, exchange questions and answers with students during the lecture Student: - Read in	A2.1; A3

Week	Topics	Objective	Activities	Exercises
[1]	[2]	[3]	[4]	[5]
			advance the next lesson - Master the basic concepts and apply to solve exercises according to the content and progress of the subject	
2	+) Absolute convergence, conditional convergence +) Tests for absolute convergence: Ratio (D'Alembert's) test, Root (Cauchy's) test +) Alternating series: Leibniz's test 3. Properties of absolutely/conditionally convergent series +) Algebraic sum of two convergent series. +) Changing order of terms and product of absolute convergent series.		Lecturer - Lecture, exchange questions and answers with students during the lecture Student: - Read in advance the next lesson - Master the basic concepts and apply to solve exercises according to the content and progress of the subject	A2.1; A3
3	**II. Series of real functions, power series** (5+5) 1. Series of real functions, domain of pointwise convergence, sum of series 2. Uniform convergence, Cauchy and Weierstrass tests for uniform convergence 3. Properties of uniformly convergent series of functions: Continuity, Differentiability, Integrability			A2.1; A3
4	4. Power series: Abel's theorem, radius of convergence, interval of convergence 5. Functions given by power series: term-wise limit, term-wise differentiation and integration			A2.1; A3
5	6. Representation of functions by power series: Taylor series, Maclaurin series, some important Taylor series. **III. Fourier series** 1. Trigonometric series, Fourier coefficients and series of a piecewise-continuous periodic function 2. Representation of functions by Fourier series of $2l$ -periodic functions:			A2.1; A3

Week	Topics	Objective	Activities	Exercises
[1]	[2]	[3]	[4]	[5]
	Dirichlet's Theorem			
6	3. Fourier series of odd or even functions 4. Half-range Fourier series of periodic functions defined on half-interval $(0, l)$.			A2.2; A3
7	**PART TWO: DIFFERENTIAL EQUATIONS (17+17)** **I. Introduction to differential equations, basic concepts** 1. Natural decay and growth, Mathematical models 2. Differential equations: definitions, orders, linear equations, solutions. **II. First order differential equations** 1. Separable equations			A2.2; A3
8	2. Homogeneous equations 3. Exact equations 4. Integrating factor 5. Linear equations 6. Bernoulli equations			A2.2; A3
9	7. Modelling with linear first order equations: Electric circuits, RL-circuits 8. Existence and Uniqueness theorem. **III. Homogeneous second order linear equations** 1. Linear free oscillation models: mass-spring systems. Homogeneous linear equations	M1.1 M1.2 M2.1 M2.2 M2.3	- Lecture, exchange questions and answers with students during the lecture Student: - Read in advance the next lesson - Master the basic concepts and apply to solve exercises according to the content and progress of the subject	A2.2; A3
10	2. Existence and Uniqueness theorem 3. Linearity, Linearly independent solutions, Wronskian, Structure of solutions to homogeneous equations			A2.2; A3
11	4. Homogeneous linear equations with constant coefficients: Characteristic equations, general solutions 5. Modeling (revisited): Free Oscillation Mass-Spring problem (damped and undamped oscillations). **IV. Nonhomogeneous linear equations** 1. Forced oscillation models: Mass-Spring systems with external forces	M1.1 M1.2 M2.1 M2.3		A3

Week	Topics	Objective	Activities	Exercises
[1]	[2]	[3]	[4]	[5]
12	2. General solutions for nonhomogeneous linear equations: Structure of solutions, method of undetermined coefficients 3. Method of variation of parameters 4. Forced oscillation: Mass-Spring systems, solutions and resonance.			A3
13	**V. Laplace transform and applications** 1. Unilateral Laplace transform for piecewise-continuous functions: Definition and Domain of definitions 2. Inverse Laplace transform 3. Linear properties.			A3
14	4. Tables of Laplace transform 5. Convolution, properties, Laplace transform of convolutions			A3
15	6. Applications of Laplace transforms to solve initial value problems +) The subsidiary equations, the diagram for solving initial value problems +) The transfer functions +) The higher order linear differential equations.			A3
16	Summary - Revision	M1, M2		A3

7. RULES OF THE MODULE

8. DATE OF APPROVAL:

Faculty of Mathematics and Informatics