# MI1121E CALCULUS II

Version: 2024.1.0

## 1. GENERAL INFORMATION

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|---|---|
| **Course title:** | Calculus II |
| **Unit in charge** | Faculty of Mathematics and Informatics |
| **Course ID:** | MI1121E |
| **Course Units:** | 3(2-2-0-6)<br>- Lecture: 30 hours<br>- Seminar: 30 hours |
| **Previous module:** | MI1111E Calculus I |
| **Prerequisites:** | MI1111E Calculus I |
| **Companion module:** | MI1131E Calculus III |

## 2. DESCRIPTION

This course provides some applications of differential calculus in geometry, the basic ideas and techniques of parameter-dependent integrals, double integrals and triple integrals, line integrals of scalar fields and vector fields, surface integrals of scalar fields and vector fields, and vector fields.

## 3. OBJECTIVES AND EXPECTED OUTCOMES

Students who complete this module have the abilities to:

| Objectives | Objectives description/Expected Outcomes | Outcome standard allocated for modules/ Levels (I/T/U) |
|---|---|---|
| [1] | [2] | [3] |
| **M1** | **Master the basic knowledge of Caculus II and apply in practice to solve related exercises** | |
| M1.1 | Master the basic concepts such as: double integrals, triple integrals, line integrals, surface integrals, vector fields as well as applications of differential calculus | I/T |
| M1.2 | Be able to apply the knowledge to solve exercises | T/U |
| **M2** | **Achieve serious attitude and necessary skills for highly effective work** | |
| M2.1 | Be skilled at analyzing and solving problems with strong logical thinking; working independently and staying focused | T/U |
| M2.2 | Identify some practical problems that can be solved by using tools of calculus. | I/T/U |
| M2.3 | Gain serious working attitude, proactive creativity, adaptation to highly competitive working environment | I/T |

## 4. COURSE MATERIALS

### Textbooks


[1] James Stewart (2016). *Calculus: Concepts and Contexts, eighth edition*. Thomson, Brooks/Cole Publishing Company

[2] Nguyễn Đình Trí, Trần Việt Dũng, Trần Xuân Hiển, Nguyễn Xuân Thảo (2015). *Toán học cao cấp tập 2.* NXB Giáo dục.


### References


[1] Trần Bình (2005). *Giải tích II*. NXB Khoa học và Kỹ thuật.

[2] Nguyễn Đình Trí, Trần Việt Dũng, Trần Xuân Hiển, Nguyễn Xuân Thảo (2017). *Bài tập Toán học cao cấp tập 2.* NXB Giáo dục.

[3] Trần Thị Kim Oanh, Phan Xuân Thành, Lê Chí Ngọc, Nguyễn Thị Thu Hương, *Giải tích II: Hàm số nhiều biến số (bài giảng dành cho sinh viên các trường kĩ thuật)*, NXB Bách Khoa Hà Nội, 2022.

[4] Khoa Toán – Tin (2023), *Slides bài giảng Giải tích 2* (tài liệu lưu hành nội bộ).


## 5. ASSESSMENT

| Components | Evaluation method | Description | Rated outcome standards | Proportion |
|---|---|---|---|---|
| [1] | [2] | [3] | [4] | [5] |
| **A1. Attendance point** | Learning attitude and attendance of the students during the course | Learning attitude of the students | | **20%** |
| **A2. Periodic test mark (*)** | **A2.1. 1st periodic test** (KT1 mark, 15 scale) (Content: From the 1st week to the 5th week) | Quizzes | M1.1, M1.2, M2.1, M2.2, M2.3 | **30%** |
| | **A2.2. 2nd periodic test** (KT2 mark, 15 scale) (Content: From the 6th week to the 10th week) | | | |
| **A3. Final exam mark** | **Final exam** | Essay | M1.1, M1.2, M2.1, M2.2, M2.3 | **50%** |

*(*) Periodic test mark (DKTDK) is calculated according to the formula DKTDK =1/3(KT1+KT2) and will be adjusted by adding points for the performance of students during the course which vary from –1 to +1 according to the Rule of Faculty of Mathematics and Informatics accompanied with the Regulations of Higher Education of Hanoi University of Science and Technology.*

## 6. COURSE PLAN

| Week | Topics | Objective | Activities | Exercises |
|---|---|---|---|---|
| [1] | [2] | [3] | [4] | [5] |
| 1 | **Chapter 1: Applications of differential calculus in geometry**<br>1.1. Applications in plane geometry<br>- Normal vector and equations for tangent lines and normal lines of a curve at a point.<br>- Curvature: mean curvature, curvature at a point, formula of curvature at a point (no proof) and examples.<br>- Envelope of a family of parametric curves: definition, formula, examples.<br>1.2. Applications in spatial geometry<br>- Vector functions, derivative of vector functions ($\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}$) and properties. | M1.1<br>M1.2<br>M2.1<br>M2.2<br>M2.3 | Lecturer:<br>- Self-introduce<br>- Introduce the course outline<br>- Explain teaching and learning methods; and forms of subject assessment<br>- Lecture, exchange questions and answers with students during the lecture<br>Student:<br>- Read in advance the next lesson<br>- Master the basic concepts and apply to solve exercises according to the content and progress of the subject | A1, A2.1, A3 |
| 2 | - Curves: equations of tangent lines and normal planes at a point of curves, curvature at a point of curves (formulas).<br>- Surfaces: equations of tangent planes and normal lines at a point of surfaces (formulas).<br>**Chapter 2. Multiple integrals**<br>2.1. Double integrals<br>- Definition, geometric meaning, properties.<br>- Calculations of double integrals in the Cartesian coordinate system. | M1.1<br>M1.2<br>M2.1<br>M2.2<br>M2.3 | Lecturer<br>- Lecture, exchange questions and answers with students during the lecture<br>Student:<br>- Read in advance the next lesson<br>- Master the basic concepts | A1, A2.1, A3 |
| 3 | - Change of variables in double | M1.1 | | A1, A2.1, |

| Week | Topics | Objective | Activities | Exercises |
|---|---|---|---|---|
| [1] | [2] | [3] | [4] | [5] |
| | integrals: general change of variables formula, change of variables in polar coordinate system. | M1.2<br>M2.1<br>M2.2<br>M2.3 | and apply to solve exercises according to the content and progress of the subject | A3 |
| 4 | - Applications of double integrals: Calculate the volume of an object, the area of a plane domain, the area of a surface (formulas and examples).<br>2.2. Triple integrals<br>- Definition, geometric meaning, properties. | M1.1<br>M1.2<br>M2.1<br>M2.2<br>M2.3 | | A1, A2.1, A3 |
| 5 | - Calculations of triple integrals in the Cartesian coordinate system.<br>- Change of variables in triple integrals: general change of variables formula, change of variables in cylindrical coordinate system, change of variables in spherical coordinate system. | M1.1<br>M1.2<br>M2.1<br>M2.2<br>M2.3 | | A1, A2.1, A3 |
| 6 | - Applications: Calculate the volume of an object.<br>**Chapter 3. Parameter Dependent Integrals**<br>3.1. Definite Integrals depending on parameters<br>- Definition<br>- Theorems on continuity. | M1.1<br>M1.2<br>M2.1<br>M2.2<br>M2.3 | | A1, A2.2, A3 |
| 7 | - Theorems on differentiation under integral sign, integration under integral sign.<br>3.2. Improper Integrals depending on parameters<br>- Definition<br>- Uniform convergence, Weierstrass theorem. | M1.1<br>M1.2<br>M2.1<br>M2.2<br>M2.3 | | A1, A2.2, A3 |
| 8 | - Properties: continuity, differentiation under integral sign, integration under integral sign.<br>3.3. Euler's integrals<br>- Introduce Gamma function ($\Gamma(p)$) and properties: definiteness, continuity, infinite differentiability. | M1.1<br>M1.2<br>M2.1<br>M2.2<br>M2.3 | | A1, A2.2, A3 |

| Week | Topics | Objective | Activities | Exercises |
|---|---|---|---|---|
| [1] | [2] | [3] | [4] | [5] |
| | $\Gamma(p+1) = p\Gamma(p) \quad \forall p > 0,$<br>$\Gamma(p)\Gamma(1-p) = \frac{\pi}{\sin(p\pi)} \quad (0 < p < 1)$<br>(no proof).<br>- Beta function: Introduce Beta function ($\mathrm{B}(p,q)$) with its two types and properties (no proof): symmetry.<br>$\mathrm{B}(p,q) = \frac{p}{p+q-1}\mathrm{B}(p,q-1),$<br>$\mathrm{B}(p,q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}.$ | | | |
| 9 | **Chapter 4. Line Integrals**<br>4.1. Line integrals of scalar fields<br>- Definition<br>- Calculation<br>4.2. Line integrals of vector fields<br>- Definition, physical meaning.<br>- Properties | M1.1<br>M1.2<br>M2.1<br>M2.2<br>M2.3 | - Lecture, exchange questions and answers with students during the lecture<br>Student:<br>- Read in advance the next lesson<br>- Master the basic concepts and apply to solve exercises according to the content and progress of the subject | A1, A2.2, A3 |
| 10 | - Relation of line integrals of scalar fields and line integrals of vector fields.<br>- Calculation<br>- Green's Theorem (proof for the case of a simple region). | M1.1<br>M1.2<br>M2.1<br>M2.2<br>M2.3 | | A1, A2.2, A3 |
| 11 | - Path independence of line integrals (no proof); find a function $u(x,y)$ such that $du = Pdx + Qdy$.<br>**Chapter 5. Surface integrals**<br>5.1 Surface integrals of scalar fields<br>- Definition<br>- Calculation | M1.1<br>M1.2<br>M2.1<br>M2.2<br>M2.3 | | A1, A3 |
| 12 | 5.2 Surface integrals of vector fields<br>- Definition, properties.<br>- Relation of surface integrals of scalar fields and surface integrals of vector fields.<br>- Calculation | M1.1<br>M1.2<br>M2.1<br>M2.2<br>M2.3 | | A1,<br>A3 |
| 13 | - Ostrogradsky's Theorem, Stoke's Theorem (no proof).<br>**Chapter 6.** Field Theory<br>6.1 Scalar Fields | M1.1<br>M1.2<br>M2.1<br>M2.2 | | A1,<br>A3 |

| Week | Topics | Objective | Activities | Exercises |
|---|---|---|---|---|
| [1] | [2] | [3] | [4] | [5] |
| | - Notions of scalar fields and level surfaces.<br>- Directional derivative: Definition, Theorem on relation between directional derivative and partial derivative. | M2.3 | | |
| 14 | - Gradient: Definition of vector $\text{grad}u$ and theorem $\frac{\partial u}{\partial \vec{\ell}} = ch_{\vec{\ell}}\text{grad}u$ (no proof) and properties.<br>6.2 Vector Fields<br>- Notions of vector fields and flow lines, system of differential equations of flow lines.<br>- The flux, div, incompressible fields: the flux of a vector field across oriented surface *S*, div (divergence), properties, incompressible fields, source (point), sink (point). | M1.1<br>M1.2<br>M2.1<br>M2.2<br>M2.3 | | A1, A3 |
| 15 | - Circulation and curl vector: the circulation of a vector field around an oriented closed curve, curl vector, curly point.<br>- Conservative vector fields: notions of conservative vector fields $\vec{F}$, the potential function for $\vec{F}$, conditions for a vector field to be conservative, conditions for an expression to be the total differential, path independence of spatial line integrals. | M1.1<br>M1.2<br>M2.1<br>M2.2<br>M2.3 | | A1, A3 |
| 16 | **Summary** | | | A1, A3 |

**7. RULES OF THE MODULE**

(Regulations of the course if any)

**8. DATE OF APPROVAL: …….**

**Faculty of Mathematics and Informatics**