On the Hartley−Fourier cosine and Hartley−Fourier sine generalized convolution

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Abstract In this paper we construct and study a new generalized convolution $(f * g)(x)$ of functions f, g for the Hartley (H_1, H_2) and the Fourier sine (F_s) integral transforms. We will show that these generalized convolutions satisfy the following factorization equalities:

$$H_{\left\{ {1 \atop 2} \right\}}(f * g)(y) = \pm (F_s f)(y)(H_{\left\{ {2 \atop 1} \right\}} g)(y), \quad \forall y \in \mathbb{R}.$$

We prove the existence of this generalized convolution on different function spaces, such as $L_1(\mathbb{R}), L_p^{\alpha,\beta,\gamma}(\mathbb{R})$. As examples, applications to solve a type of integral equations and a type of systems of integral equations are presented.

Similarly, we study a new generalized convolution

$$(f \underset{1}{*} g)(x) = \frac{1}{2\pi} \int_0^\infty [g(x+u) + g(x-u)]\, f(u)\, du, \quad x \in \mathbb{R}, \tag{0.1}$$

of two functions $f \in L_1(\mathbb{R}_+)$ and $g \in L_1(\mathbb{R})$. The convolution (0.1) differs from the convolution Fourier cosine by being considered on the whole real line. We show that the convolution (0.1) is related to the Hartley transform H_i $(i = 1, 2)$ and the Fourier cosine transform F_c by the factorization equality

$$H_i(f \underset{1}{*} g)(y) = (F_c f)(y)(H_i g)(y), \quad \forall y \in \mathbb{R} \ (i = 1, 2). \tag{0.2}$$

As applications we obtain solutions in closed form for some special cases of the Toeplitz plus Hankel integral equation on the whole real line with the help of a new Hartley–Fourier cosine generalized convolution.

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