Recovery of the heat equation from a single boundary measurement

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Consider the heat process in a finite length rod, where the heat source is proportional to the temperature distribution

(1)
$$\begin{cases} u_t(x,t) = u_{xx}(x,t) - q(x)u(x,t), & 0 < x < 1, \quad t > 0, \\ u(0,t) = 0, \quad u(1,t) = a(t), \\ u(x,0) = 0. \end{cases}$$

We are concerned with the recovery of the heat coefficient q(x) from the measurement of the heat flux $u_x(1,t) = b(t)$, at one end of the rod only.

We show that if u(1,t) = a(t) is a nontrivial, nonnegative, and continuously differentiable function with compact support on (0,T), then a single measurement of $u_x(1,t) = b(t)$ either on (T,T_1) or at $t = 1, 2, \cdots$ determines q(x) uniquely.

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